Security in Process Calculi

Service Oriented Architectures

Module 1 – Basic technologies

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Overview

Pi calculus

- Core language for parallel programming
- Modeling security via name scoping

Applied pi calculus

- Modeling cryptographic primitives with functions and equational theories
- Equivalence-based notions of security
- A little bit of operational semantics
- Security as testing equivalence

Pi Calculus

[Milner et al.]

Fundamental language for concurrent systems

- High-level mathematical model of parallel processes
- The “core” of concurrent programming languages
- By comparison, lambda-calculus is the “core” of functional programming languages

Mobility is a basic primitive

- Basic computational step is the transfer of a communication link between two processes
- Interconnections between processes change as they communicate

Can be used as a general programming language

A Little Bit of History

1980: Calculus of Communicating Systems (CCS) [Milner]

1992: Pi Calculus [Milner, Parrow, Walker]

- Ability to pass channel names between processes

1998: Spi Calculus [Abadi, Gordon]

- Adds cryptographic primitives to pi calculus
- Security modeled as scoping
- Equivalence-based specification of security properties
- Connection with computational models of cryptography

2001: Applied Pi Calculus [Abadi, Fournet]

- Generic functions, including crypto primitives

Pi Calculus Syntax

Terms

- M, N ::= x — *variables*
  | n — *names*

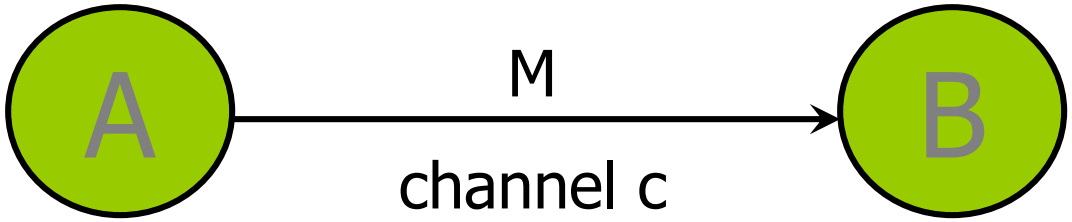
} *Let u range over names and variables*

Processes

- P,Q ::= nil — *empty process*
  | $\bar{u}\langle N\rangle.P$ — *send term N on channel u*
  | u(x).P — *receive term from channel P and assign to x*
  | !P — *replicate process P*
  | P|Q — *run processes P and Q in parallel*
  | $(\nu n)P$ — *restrict name n to process P*

Modeling Secrecy with Scoping

A sends M to B over secure channel c



$$A(M) = \bar{c}\langle M \rangle$$
$$B = c(x).nil$$
$$P(M) = (\nu c)(A(M) \mid B)$$

This restriction ensures that channel c is "invisible" to any process except A and B (other processes don't know name c)

Secrecy as Equivalence

$$A(M) = \bar{c}\langle M \rangle$$
$$B = c(x).nil$$
$$P(M) = (\nu c)(A(M) \mid B)$$

Without (νc), attacker could run process c(x) and tell the difference between P(M) and P(M')

P(M) and P(M') are "equivalent" for any values of M and M'

- No attacker can distinguish P(M) and P(M')

Different notions of "equivalence"

- Testing equivalence or observational congruence
- Indistinguishability by any probabilistic polynomial-time Turing machine

Another Formulation of Secrecy

$$A(M) = \bar{c}\langle M \rangle$$
$$B = c(x).nil$$
$$P(M) = (\nu c)(A(M) \mid B)$$

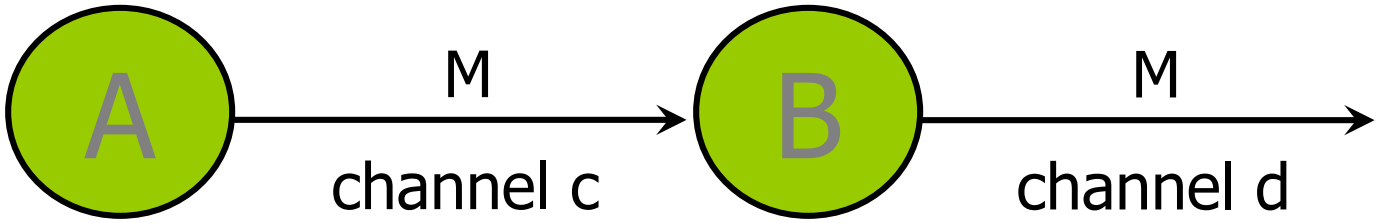
No attacker can learn name n from P(n)

- Let Q be an arbitrary attacker process, and suppose it runs in parallel with P(n)
- **For any process Q in which n does not occur free, P(n) | Q will never output n**

Modeling Authentication with Scoping

A sends M to B over secure channel c

B announces received value on public channel d



$$A(M) = \bar{c}\langle M \rangle$$

$$B = c(x).\bar{d}\langle x \rangle$$

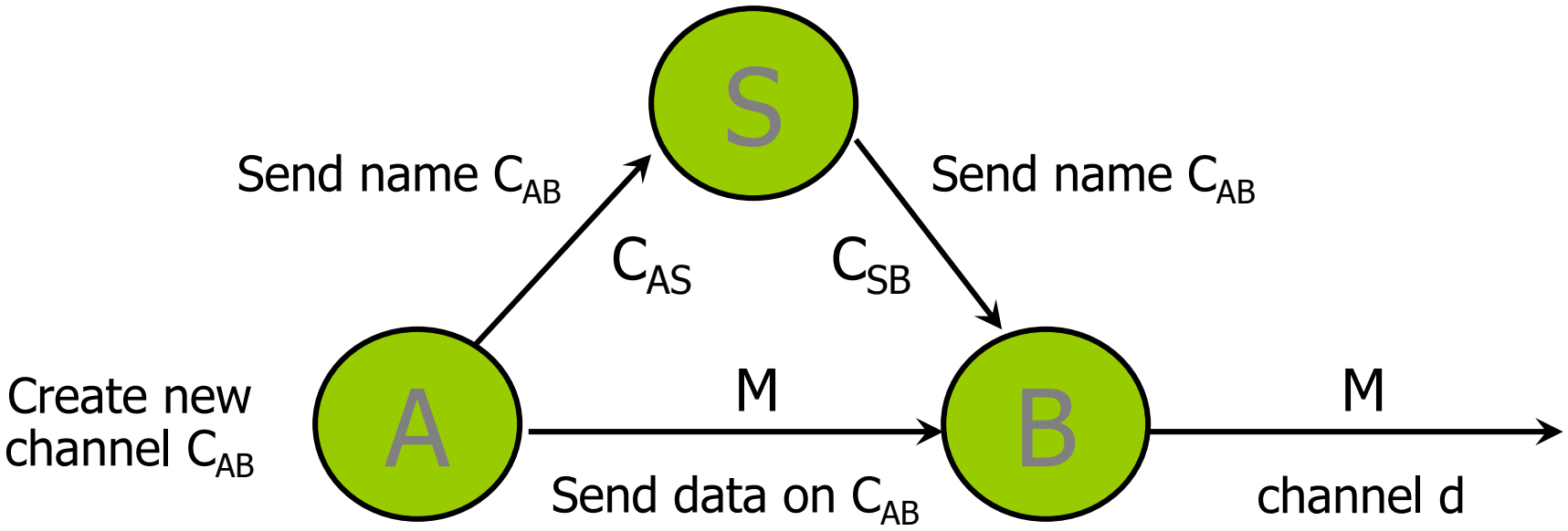
$$P(M) = (\nu c)(A(M) \mid B)$$

Specifying Authentication

$$A(M) = \bar{c}\langle M\rangle$$
$$B = c(x).\bar{d}\langle x\rangle$$
$$P(M) = (\nu c)(A(M) \mid B)$$

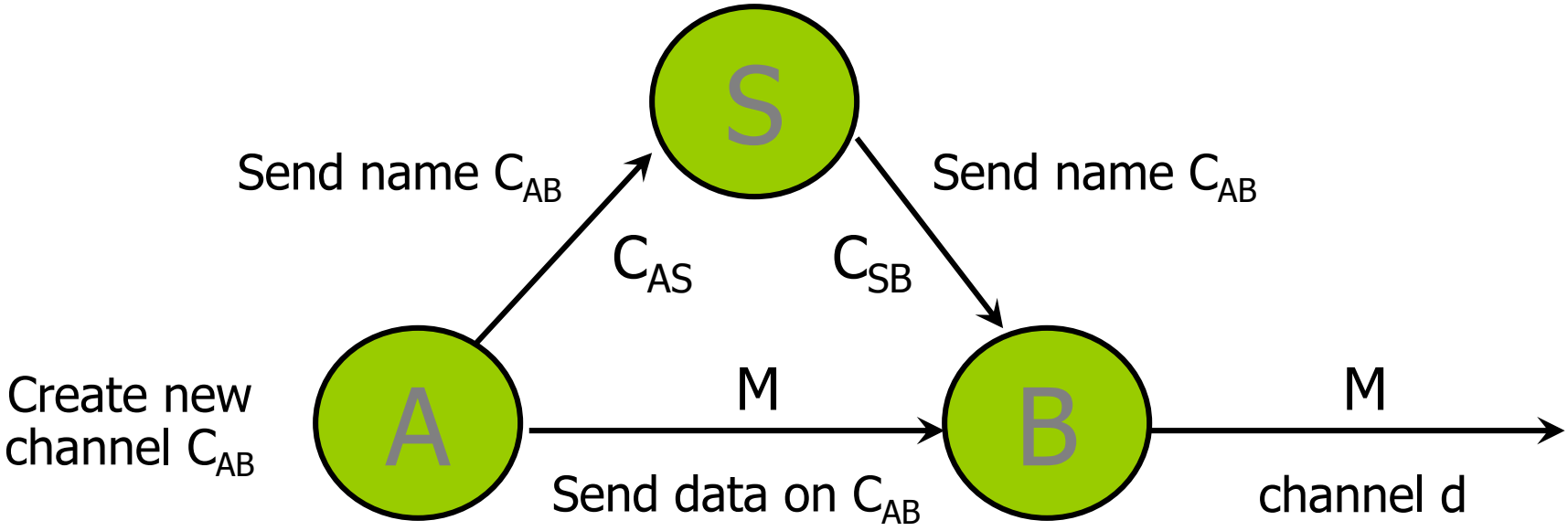
For any value of M, if B outputs M on channel d, then A previously sent M on channel c

A Key Establishment Protocol



1. **A and B have pre-established pairwise keys with server S**
   - Model these keys as names of pre-existing communication channels
2. **A creates a new key and sends it to S, who forwards it to B**
   - Model this as creation of a new channel name
3. **A sends M to B encrypted with the new key, B outputs M**

Key Establishment in Pi Calculus



$$A(M) = (\nu c_{AB})\,\overline{c_{AS}}\langle c_{AB}\rangle.\,\overline{c_{AB}}\langle M\rangle$$

$$S = c_{AS}(x).\,\overline{c_{SB}}\langle x\rangle$$

$$B = c_{SB}(x).x(y).\,\overline{d}\langle y\rangle$$

Note communication on a channel with a dynamically generated name

$$P(M) = (\nu c_{AS})\,(\nu c_{SB})\,(A(M) \mid B \mid S)$$

Applied Pi Calculus

In pi calculus, channels are the only primitive

This is enough to model some forms of security

- Name of a communication channel can be viewed as an "encryption key" for traffic on that channel
  - –A process that doesn't know the name can't access the channel
- Channel names can be passed between processes
  - –Useful for modeling key establishment protocols

To simplify protocol specification, applied pi calculus adds functions to pi calculus

- Crypto primitives modeled by functions and equations

Applied Pi Calculus: Terms

M, N ::=	x	*Variable*
	| n	*Name*
	| $f(M_1,\ldots,M_k)$	*Function application*

Standard functions

- pair(), encrypt(), hash(), …

Simple type system for terms

- Integer, Key, Channel⟨Integer⟩, Channel⟨Key⟩

Applied Pi Calculus: Processes

P,Q ::= nil	*empty process*
| $\bar{u}\langle N\rangle.P$	*send term N on channel u*
| u(x).P	*receive from channel P and assign to x*
| !P	*replicate process P*
| P|Q	*run processes P and Q in parallel*
| $(\nu n)P$	*restrict name n to process P*
| if M = N then P else Q	*conditional*

Modeling Crypto with Functions

Introduce special function symbols to model cryptographic primitives

Equational theory models cryptographic properties

Pairing

- Functions pair, first, second with equations:

  first(pair(x,y)) = x

  second(pair(x,y)) = y

Symmetric-key encryption

- Functions symenc, symdec with equation:

  symdec(symenc(x,k),k)=x

More Equational Theories

Public-key encryption

- Functions pk,sk generate public/private key pair pk(x),sk(x) from a random seed x
- Functions pdec,penc model encryption and decryption with equation:

  pdec(penc(y,pk(x)),sk(x)) = y
- Can also model "probabilistic" encryption:

  pdec(penc(y,pk(x),z),sk(x)) = y

Models random salt
(necessary for semantic security)

Hashing

- Unary function hash with no equations
- hash(M) models applying a one-way function to term M

Yet More Equational Theories

Public-key digital signatures

- As before, functions pk,sk generate public/private key pair pk(x),sk(x) from a random seed x
- Functions sign,verify model signing and verification with equation:

$$verify(y,sign(y,sk(x)),pk(x)) = y$$

XOR

- Model self-cancellation property with equation:

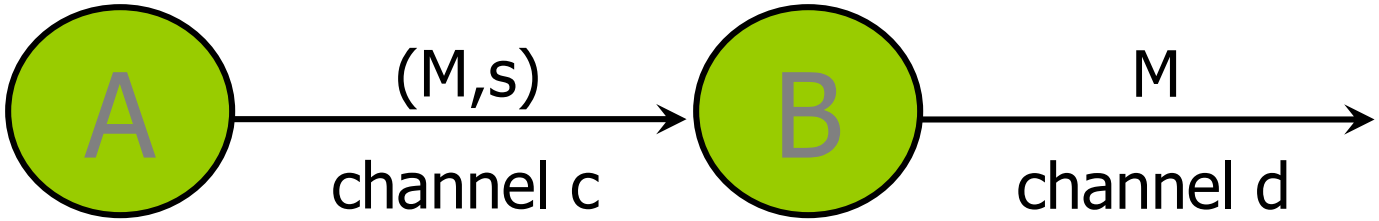
$$xor(xor(x,y),y) = x$$

- Can also model properties of cyclic redundancy codes:

$$crc(xor(x,y)) = xor(crc(x),crc(y))$$

Dynamically Generated Data

Use built-in name generation capability of pi calculus to model creation of new keys and nonces



$$A(M) = \bar{c}\langle(M,s)\rangle$$

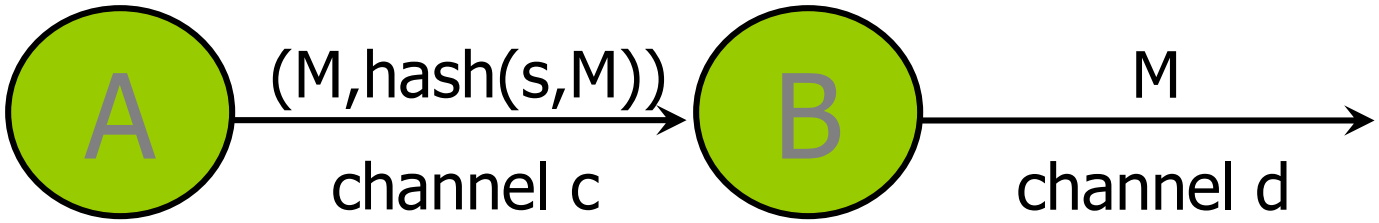
$$B = c(x).\text{if } second(x)=s \text{ then } \bar{d}\langle first(x)\rangle$$

$$P(M) = (\nu s)(A(M) \mid B)$$

Models creation of fresh capability every time A and B communicate

capability s may be intercepted!

Better Protocol with Capabilities



Hashing protects integrity of M and secrecy of s

$$A(M) = \bar{c}\langle(M,\mathtt{hash}(s,M))\rangle$$

$$B = c(x).\mathtt{if}\ \mathtt{second}(x) = \mathtt{hash}(s,\mathtt{first}(x))\ \mathtt{then}\ \bar{d}\langle\mathtt{first}(x)\rangle$$

$$P(M) = (\nu s)(A(M) \mid B)$$

Proving Security

"Real" protocol

- Process-calculus specification of the actual protocol

"Ideal" protocol

- Achieves the same goal as the real protocol, but is secure by design
- Uses unrealistic mechanisms, e.g., private channels
- Represents the desired behavior of real protocol

To prove the real protocol secure, show that no attacker can tell the difference between the real protocol and the ideal protocol

- Proof will depend on the model of attacker observations

Example: Challenge-Response

Challenge-response protocol

$A \rightarrow B \quad \{i\}_k$

$B \rightarrow A \quad \{i+1\}_k$

This protocol is secure if it is indistinguishable from this "ideal" protocol

$A \rightarrow B \quad \{random_1\}_k$

$B \rightarrow A \quad \{random_2\}_k$

Example: Authentication

Authentication protocol

$A \rightarrow B$ $\{i\}_k$

$B \rightarrow A$ $\{i+1\}_k$

$A \rightarrow B$ "Ok"

This protocol is secure if it is indistinguishable from this "ideal" protocol

$A \rightarrow B$ $\{random_1\}_k$

$B \rightarrow A$ $\{random_2\}_k$

$B \rightarrow A$ $random_1$, $random_2$ on a magic secure channel

$A \rightarrow B$ "Ok" if numbers on real & magic channels match

Security as Observational Equivalenc

Need to prove that two processes are observationally equivalent to the attacker

Complexity-theoretic model

- Prove that two systems cannot be distinguished by any probabilistic polynomial-time adversary

[Beaver '91, Goldwasser-Levin '90, Micali-Rogaway '91]

Abstract process-calculus model

- Cryptography is modeled by abstract functions
- Prove testing equivalence between two processes
- Proofs are easier, but it is nontrivial to show computational completeness [Abadi-Rogaway '00]

Structural Equivalence

$$P \mid \mathrm{nil} \equiv P$$

$$P \mid Q \equiv Q \mid P$$

$$P \mid (Q \mid R) \equiv (P \mid Q) \mid R$$

$$!P \equiv P \mid !P$$

$$(\nu m)\,(\nu n)P \equiv (\nu n)\,(\nu m)P$$

$$(\nu n)\mathrm{nil} \equiv \mathrm{nil}$$

$$(\nu n)(P \mid Q) \equiv P \mid (\nu n)Q \quad \textit{if } n \textit{ is not a free name in } P$$

$$P[M/x] \equiv P[N/x] \quad \textit{if } M=N \textit{ in the equational theory}$$

Operational Semantics

Reduction $\rightarrow$ is the smallest relation on closed processes that is closed by structural equivalence and application of evaluation contexts such that

$$\bar{a}\langle M\rangle.P \mid a(x).Q \;\rightarrow\; P \mid Q[M/x]$$

models P sending M to Q on channel a

$$\text{if } M = M \text{ then } P \text{ else } Q \;\rightarrow\; P$$

$$\text{if } M = N \text{ then } P \text{ else } Q \;\rightarrow\; Q$$

for any ground M, N s.t. $M \neq N$ in the equational theory

Equivalence in Process Calculus

Standard process-calculus notions of equivalence such as bisimulation are not adequate for cryptographic protocols

- Different ciphertexts leak no information to the attacker who does not know the decryption keys

$(\nu k)\overline{c}\langle symenc(M,k)\rangle$ and $(\nu k)\overline{c}\langle symenc(N,k)\rangle$ send different messages, but they should be treated as equivalent when proving security

- In each case, a term is encrypted under a fresh key
- No test by the attacker can tell these apart

Testing Equivalence

Informally, two processes are equivalent if no environment can distinguish them

A test is a process R and channel name w

- Informally, R is the environment and w is the channel on which the outcome of the test is announced

A process P passes a test (R,w) if P | R may produce an output on channel w

- There is an interleaving of P and R that results in R being able to perform the desired test

Two processes are equivalent if they pass the same tests

Advantages and Disadvantages

Proving testing equivalence is hard

- Need to quantify over all possible attacker processes and all tests they may perform
- There are some helpful proof techniques, but no fully automated tools and very few decision procedures

Testing equivalence is a congruence

- Can compose protocols like building blocks

Equivalence is the "right" notion of security

- Direct connection with definitions of security in complexity-theoretic cryptography
- Contrast this with invariant- and trace-based definitions

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